Newton Chinese Language School

Math 7 Final Exam 6/17/2012 **Name:** _____

Multiple Choice Questions (3 points each, 75 points total)

1. Factor $3x^2 - 17x - 6$. Answer: $(3x+1)(x-6)$
2. Factor $6y^2 - 29y - 5$. Answer: $(6y+1)(y-5)$
3. Which of the following equations cannot be solved by factoring with integer coefficients?
   A) $12x^2 - 15x - 63 = 0$ B) $12x^2 + 46x - 8 = 0$
   C) $6x^2 - 38x - 28 = 0$ D) $8x^2 - 49x - 68 = 0$

   b^2-4ac must be a full square
4. Which of the following is a correct factorization of $72x^2 - 24x + 2$?
   A) $-9(3x-1)^2$ B) $8\left(9x-\frac{1}{2}\right)^2$ C) $8(3x-\frac{1}{2})(3x-\frac{1}{2})$ D) $-8\left(3x-\frac{1}{2}\right)^2$
5. Factor $x^3 - 5x^2 + 4x - 20$. Answer: $(x-5)(x^2+4)$ $x^2(x-5)+4(x-5)=(x-5)(x^2+4)$
6. $(4x-9)(7x-2) =$ $28x^2-71x+18$
7. $(-x^2 - 5x + 3) - (7x^2 - x - 5) =$ $-8x^2-4x+8$
8. Divide $12x^2 - 20x + 8$ by $4x$. Answer: $3x-5+\frac{2}{x}$
9. Solve the equation: $\frac{x}{6}+\frac{15}{x}=\frac{5}{6}$. $x =$ no answer
10. Simply the expression $\frac{x^3-10x^2+9x}{x^2+5x-6}$. Answer: $\frac{x(x-9)}{x+6}$
11. 18.6 miles is 15% of what distance? Answer: 124
12. Which of the following equations models inverse variation?
    A) $y=\frac{x}{43}$ B) $y = 0.3x$ C) $y = x + 5$ D) $xy = 0.1$ E) $y = 0.34 - x$
13. The variables x and y vary inversely. When x is 9, y is 1/3. If x is 3, what is y? Answer: 1
14. Which trinomial represents the area of a trapezoid whose top side is $(2x + 1)$, bottom side is $(3x + 3)$, and height is $(x + 3)$? The area is $A = \frac{1}{2}h(b1 + b2)$.
    A) $\frac{5}{2}x^2 + 19x + 12$ B) $5x^2 + 19x + 6$ C) $\frac{5}{2}x^2 + \frac{19}{2}x + 6$ D) $5x^2 + 19x - 6$ E) $\frac{5}{2}x^2 + \frac{17}{2}x + 6$
15. What are the coordinate of the vertex of the graph of $y = (x - 6)(x + 5)$?
    A) $(-\frac{1}{2}, -24\frac{1}{2})$ B) $(\frac{1}{2}, -25\frac{1}{2})$ C) $(2, -28)$ D) $(\frac{1}{2}, -30\frac{1}{4})$ E) $(-\frac{1}{2}, -24\frac{1}{4})$
16. Find the LCD of $\frac{7x}{x^2-9}$ and $\frac{3}{x^2+x-6}$.
    A) $(x+3)(x-3)$ B) $(x^2-9)(x^2+x-6)$ C) $(x+3)(x-2)$ D) $(x+3)(x-3)(x-2)$
    E) $(x-3)(x-2)$
17. Evaluate the expression $5\sqrt{7} + \sqrt{448} + \sqrt{175} - \sqrt{63}$.
    A) $15\sqrt{7}$ B) $16\sqrt{7}$ C) $18\sqrt{7}$ D) $20\sqrt{7}$ E) $21\sqrt{7}$

18. What is the simplified expression of $(3-\sqrt{6})^2$?

A) $9-5\sqrt{6}$ B) $3-6\sqrt{6}$ C) 15 D) $15-6\sqrt{6}$ E) None of these

19. What term should be added to $x^2-\frac{4}{7}x$ so that the result is a perfect square trinomial? Answer: $\left(\frac{2}{7}\right)^2=\frac{4}{49}$

20. In baseball, Julio averages 3 hits in every 10 at bats. What is the probability that Julio will get hits in both of his next two at bats?

A. 0.03 B. 0.09 C. 0.3 D. 0.9

21. Two people from a group of 30 will be selected at random for a prize. Twenty people in the group are women. What is the probability that both people selected will be men?

A. $\frac{3}{29}$ B. $\frac{38}{87}$ C. $\frac{56}{87}$ D. $\frac{1}{10}$

22. Ravi has 10 pairs of socks in a drawer, but none of the pairs are matched up. Each pair is a different color, and one pair is blue. Ravi has to pick his socks in the dark so he does not wake his brother. Which expression can be used to find the probability that Ravi will choose a blue sock and then a matching sock?

A. $\frac{1}{20}+\frac{1}{20}$ B. $\frac{1}{10}\cdot\frac{1}{20}$ C. $\frac{1}{20}+\frac{1}{19}$ D. $\frac{1}{10}\cdot\frac{1}{19}$

23. Which situation is best modeled by the expression ${}_{10}C_4$?

A. List's salon offers 10 kinds of shampoo and 4 kinds of conditioner. How many different ways can she shampoo and condition a client's hair?

B. Sara has a box of 10 colored pencils. How many different ways can she choose 4 colored pencils from the box?

C. A personal identification number (PIN) is a 4-digit number. Digits can be repeated in a PIN. How many different PINs are possible?

D. There are 10 chairs placed around a table. How many different ways can 4 people seat themselves around the table?

24. Your friend randomly selects 2 movies from your collection to borrow. Your collection includes 9 comedies, 5 dramas, 3 mysteries, 1 musical, and 2 horror movies. What is the probability that both movies are comedies?

A. $\frac{1}{36}$ B. $\frac{18}{95}$ C. $\frac{81}{400}$ D. $\frac{2}{9}$ E. $\frac{331}{380}$

25. What is the value of $\frac{(n+1)!}{(n-1)!}$?

A. n^2+n B. $2n+1$ C. $n!$ D. n^2-1 E. -1

Solve problems below with steps (5 points each, 25 points total)

26. A stadium's seating is arranged as follows: the first row has 15 seats. Starting from the second row, every row has two more seats than the previous row. Can you represent n^{th} row? How many people can 10^{th} row sit?

$a_n = 15+2(n-1)$

$a_{10} = 15+2\cdot 9 = 15+18 = 33$

27. Given an arithmetic sequence $\{a_n\}$, S_n is the sum of the first n terms. Prove that S_6, $S_{12} - S_6$, $S_{18} - S_{12}$ also make an arithmetic sequence.

1st item is a, common difference is d,

then $b_1 = S_6 = \frac{6(a_1+a_6)}{2} = \frac{6(a+a+5d)}{2} = 3(2a+5d) = 6a+15d$

$b_2 = S_{12} - S_6 = \frac{12(a_1+a_{12}) - 6(a_1+a_5)}{2} = \frac{12(a+a+11d) - 6(a+a+5d)}{2} = \frac{24a+12\cdot 11d - 12a - 30d}{2} = 6a+51d$

$b_3 = S_{18} - S_{12} = \frac{18(a+a+17d) - 12(a+a+11d)}{2} = \frac{36a+18\cdot 17d - 24a - 12\cdot 11d}{2} = 6a+87d$

$b_2 - b_1 = 51d - 15d = 36d$ $b_3 - b_2 = 87d - 51d = 36d$, because they have common difference of 36d, they make an A.S.

28. Insert 4 numbers between 160 and 5, so that they make a geometric sequence.

$160, a_2, a_3, a_4, a_5, 5$

$a_1 = 160$

$a_6 = 5 = 160 \cdot r^5$

$r^5 = \frac{1}{32}$ $r = \frac{1}{2}$,

$a_2 = 80$

$a_3 = 40$

$a_4 = 20$

$a_5 = 10$.

29. $\{a_n\}$ is a geometric sequence, S_n is the sum of its first n terms, and S_3, S_9, S_6 forms an arithmetic sequence. Prove that a_2, a_8, a_5 is also an arithmetic sequence.

$S_3 = \frac{a(r^3-1)}{r-1}$

$S_9 = \frac{a(r^9-1)}{r-1}$

$S_6 = \frac{a(r^6-1)}{r-1}$

$S_6 - S_9 = S_9 - S_3$

$\frac{2a(r^6-1-r^9+1)}{r-1} = \frac{2a(r^9-1-r^3+1)}{r-1}$

$r^6 - r^9 = r^9 - r^3$ or $r^3(r^3-r^6) = r^3(r^6-1)$

$r^3 - r^6 = r^6 - 1$

$a_2 = ar$, $a_8 = ar^7$, $a_5 = ar^4$

$a_8 - a_2 = a(r^7 - r) = ar(r^6 - 1)$

$= ar(r^3 r^6) = a(r^4 - r^7)$

$= ar^4 - ar^7 = a_5 - a_8$

a_2, a_8, a_5 make an A.S.

30. Find all values of x that satisfy: $x = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$.

$1, -x, x^2, -x^3, x^4, \cdots$ is a geometric sequence with $r = -x$,

Because it converges, $S = \frac{1}{1-r}$

$x = \frac{1}{1-(-x)} = \frac{1}{1+x}$ $x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

Because for a G.S. to convergent, $|r|$ must be smaller than 1,

so, $x = \frac{-1+\sqrt{5}}{2}$, throw away $\frac{-1-\sqrt{5}}{2}$.

Bonus Questions (10 points each)

1. $\{a_n\}$ is a geometric sequence, S_n is the sum of its first n terms. Prove that S_7, $S_{14} - S_7$, $S_{21} - S_{14}$ is also a geometric sequence.

$$b_1 = S_7 = \frac{a(r^7-1)}{r-1}$$

$$b_2 = S_{14} - S_7 = \frac{a(r^{14}-1) - a(r^7-1)}{r-1} = \frac{a(r^{14}-r^7)}{r-1} = \frac{ar^7(r^7-1)}{r-1} = r^7 \cdot b_1$$

$$b_3 = S_{21} - S_{14} = \frac{a(r^{21}-1) - a(r^{14}-1)}{r-1} = \frac{a(r^{21}-r^{14})}{r-1} = \frac{ar^{14}(r^7-1)}{r-1} = r^{14} b_1 = r^7 \cdot b_2$$

$$\frac{b_3}{b_2} = r^7, \quad \frac{b_2}{b_1} = r^7$$

S_0, S_7, $S_{14}-S_7$, $S_{21}-S_{14}$ form a G.S.

2. If a sequence is both arithmetic and geometric, prove that it is a constant sequence (every term is the same).

Say the 1st item is a, common difference is d, common ratio is r,

$a_1 = a+d$ $\quad a_1 = ar$ $\quad a+d = ar$ $\quad d = a(r-1)$

$a_2 = a+2d$ $\quad a_2 = ar^2$

$ar^2 = a+2d = a+2a(r-1) = a+2ar-2a$

$r^2 = 1+2r-2$

~~r^2-2r+1~~

$r^2-2r+1 = 0$

$(r-1)^2 = 0$

$r = 1$

$d = a(r-1) = a(1-1) = 0$

$d = 0$, it is a ~~consequence~~ constant sequence

$r = 1$, it is a constant sequence