10-5 Experimental Probability

Objectives
Determine the experimental probability of an event.

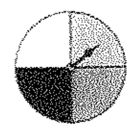
Use experimental probability to make predictions.

Vocabulary
experiment
trial
outcome
sample space
event
probability
experimental probability
prediction

Why learn this?

Experimental probability can be used by manufacturers for quality control. (See Example 4.)

An **experiment** is an activity involving chance. Each repetition or observation of an experiment is a **trial**, and each possible result is an **outcome**. The **sample space** of an experiment is the set of all possible outcomes.

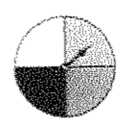
	Rolling a number cube	Tossing a coin	Spinning a game spinner
Experiment			
Sample Space	{1, 2, 3, 4, 5, 6}	{heads, tails}	{red, blue, green, yellow}

EXAMPLE 1 Identifying Sample Spaces and Outcomes

Identify the sample space and the outcome shown for each experiment.

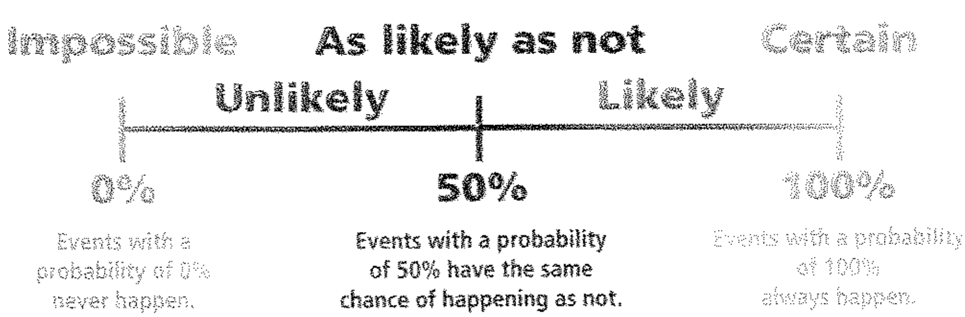
A **tossing two coins**
Sample space: {HH, HT, TH, TT}
Outcome shown: heads, tails (H, T)

B **spinning a game spinner**
Sample space: {yellow, red, blue, green}
Outcome shown: green



CHECK IT OUT! 1. Identify the sample space and the outcome shown for the experiment: rolling a number cube.

An **event** is an outcome or set of outcomes in an experiment. **Probability** is the measure of how likely an event is to occur. Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.



EXAMPLE 2 Estimating the Likelihood of an Event

Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

A **There are 31 days in August.**

August always has 31 days. This event is *certain.*

B **Carlos correctly guesses a number between 1 and 1000.**

Carlos must pick one outcome out of 1000 possible outcomes. This event is *unlikely.*

C **A coin lands heads up.**

Heads is one of two possible outcomes. This event is *as likely as not.*

D **Cecilia rolls a 10 on a standard number cube.**

A standard number cube is numbered 1 through 6. This event is *impossible.*

CHECK IT OUT! 2. Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe the event: Anthony rolls a number less than 7 on a standard number cube.

You can estimate the probability of an event by performing an experiment. The **experimental probability** of an event is the ratio of the number of times the event occurs to the number of trials. The more trials performed, the more accurate the estimate is likely to be.

Know it! Note

Experimental Probability

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

EXAMPLE 3 Finding Experimental Probability

An experiment consists of spinning a spinner. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Red	7
Blue	8
Green	5

Helpful Hint

Probabilities can be expressed as fractions, decimals, or percents. For Example 3, the probabilities are:

3A: $\frac{2}{5} = 0.4 = 40\%$

3B: $\frac{3}{4} = 0.75 = 75\%$

A **spinner lands on blue**

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{8}{7 + 8 + 5}$$

$$= \frac{8}{20}$$

$$= \frac{2}{5}$$

B **spinner does not land on green**

When the spinner does not land on green, it must land on red or blue.

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{7 + 8}{7 + 8 + 5}$$

$$= \frac{15}{20}$$

$$= \frac{3}{4}$$

CHECK IT OUT! Use the information in Example 3 to find the experimental probability of each event.

3a. spinner lands on red

3b. spinner does not land on red

You can use experimental probability to make *predictions*. A **prediction** is an estimate or guess about something that has not yet happened.

EXAMPLE 4 *Quality Control Application*

A manufacturer inspects 800 light bulbs and finds that 796 of them have no defects.

A **What is the experimental probability that a light bulb chosen at random has no defects?**

Find the experimental probability that a light bulb has no defects.

$$\frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{796}{800}$$

$$= 99.5\%$$

The experimental probability that a light bulb has no defects is 99.5%.

B **The manufacturer sent a shipment of 2400 light bulbs to a retail store. Predict the number of light bulbs in the shipment that are likely to have no defects.**

Find 99.5% of 2400.

$0.995(2400) = 2388$

The manufacturer predicts that 2388 light bulbs have no defects.

CHECK IT OUT! **4.** A manufacturer inspects 1500 electric toothbrush motors and finds that 1497 of them have no defects.

a. What is the experimental probability that a motor chosen at random will have no defects?

b. There are 35,000 motors in a warehouse. Predict the number of motors that are likely to have no defects.

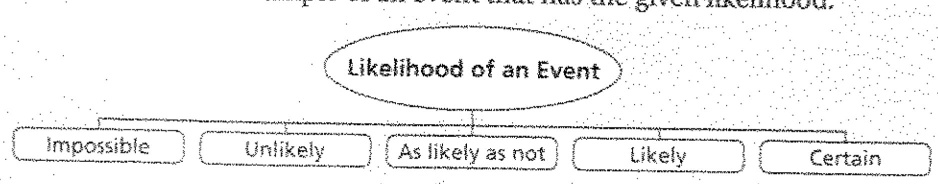
THINK AND DISCUSS

1. Explain the difference between an outcome and an event.

2. Is the experimental probability of an event always the same? Explain why or why not.

Know It! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an event that has the given likelihood.



10-5 Exercises

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GUIDED PRACTICE

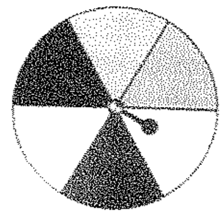
1. **Vocabulary** Give an example of an *event* that has two possible *outcomes*.

SEE EXAMPLE 1
p. 737

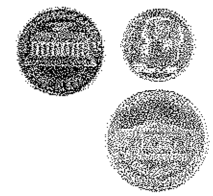
Identify the sample space and the outcome shown for each experiment.

2. rolling a number cube

3. spinning a spinner



4. tossing 3 coins



SEE EXAMPLE 2
p. 738

Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

5. Peter was born in January. Thomas was born in June. Peter and Thomas have the same birthday.
6. The football team won 9 of its last 10 games. The team will win the next game.
7. A board game has a rule that if you roll the game cube and get a 6, you get an extra turn. You get an extra turn on your first roll.

SEE EXAMPLE 3
p. 738

An experiment consists of rolling a number cube. Use the results in the table to find the experimental probability of each event.

Outcome	1	2	3	4	5
Frequency	5	6	2	2	3

8. rolling a 6
9. rolling an even number
10. not rolling a 6

SEE EXAMPLE 4
p. 739

11. **Sports** One game of bowling consists of ten frames. Elyse usually rolls 3 strikes in each game.
    a. What is the experimental probability that Elyse will roll a strike on any frame?
    b. Predict the number of strikes Elyse will roll in 18 games.

PRACTICE AND PROBLEM SOLVING

Independent Practice

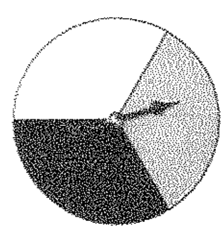
For Exercises	See Example
12–14	1
15–17	2
18–20	3
21	4

Extra Practice
Skills Practice p. S23
Application Practice p. S37

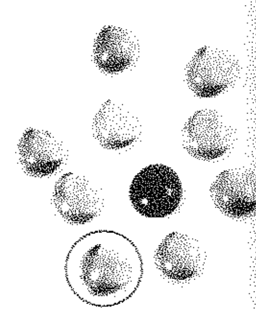
Identify the sample space and the outcome shown for each experiment.

12. tossing two coins

13. spinning a spinner



14. selecting a marble



Write *impossible, unlikely, as likely as not, likely,* or *certain* to describe each event.

15. Marlo purchased a new pair of shoes. She takes one shoe out of the box. The shoe is for the left foot.

16. Sam takes the bus to school. The bus came late twice in the last two weeks. The bus will be late today.

17. Tammy dropped two quarters on the floor. At least one of them lands heads up.

An experiment consists of randomly choosing a marble from a bag. Use the results in the table to find the experimental probability of each event.

Outcome	Frequency
Red	4
Blue	6
Green	6
Yellow	9

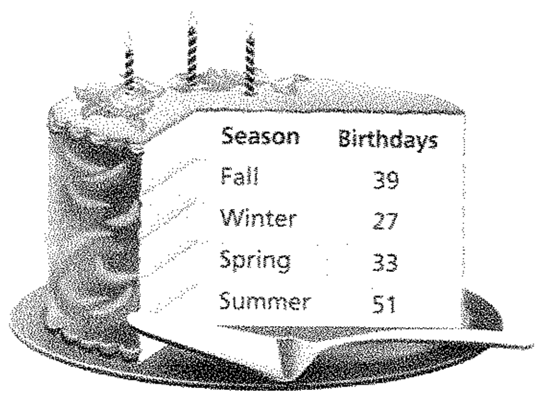
18. choosing a yellow marble

19. choosing a blue marble

20. not choosing a green marble

21. **Sports** A ski lodge inspects 80 skis and finds 4 to be defective.
    a. What is the experimental probability that a ski chosen at random will be defective?
    b. The lodge has 420 skis. Predict the number of skis that are likely to be defective.

22. The table shows the results of a survey asking students the season of their birthday. What is the experimental probability that a student has a birthday during the summer?



Season	Birthdays
Fall	39
Winter	27
Spring	33
Summer	51

23. You and your friend can either go swimming or to a movie on Thursday. The weather forecast says there is a 70% chance of rain on Thursday. Should you plan on going swimming or to a movie? Explain.

24. **Critical Thinking** Tell why it is important to repeat an experiment many times.

25. **Write About It** Explain what it means for an event to have a 50-50 chance of happening.

26. How many outcomes are in the sample space for an experiment consisting of rolling two standard number cubes?

27. **Estimation** A manufacturing company produced 986 units in one day. Of those, 9 units were found to be defective. Estimate the experimental probability that a unit produced that day was defective. Then predict approximately how many units will be defective when 5680 units are produced in one week.

MULTI-STEP TEST PREP

28. This problem will prepare you for the Multi-Step Test Prep on page 768.

In a standard deck of cards, there are 13 cards in each of four suits: hearts, diamonds, clubs, and spades. The hearts and diamonds are red and the clubs and spades are black. Ricardo randomly drew cards from a standard deck of 52 cards. The table shows the results.

Outcome	Frequency
Hearts	7
Diamonds	7
Clubs	8
Spades	6

a. Find the experimental probability of drawing a club.
b. Find the experimental probability of drawing a black suit.

TEST PREP

29. Alex rolls two standard number cubes. What is the likelihood that the sum of the numbers is less than 4?

  (A) Impossible (B) Unlikely (C) As likely as not (D) Likely

30. A community reported that $\frac{2}{3}$ of the residents had a pet and $\frac{1}{2}$ of the pet owners had a dog. If there are 84 residents in the community, how many residents are likely to have a dog?

  (F) 14 (G) 28 (H) 42 (J) 56

31. What is the probability that a number chosen at random from the list below will be a solution of the inequality $3x + 2 \leq 23$?

  $-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

  (A) $\frac{17}{18}$ (B) $\frac{8}{9}$ (C) $\frac{1}{9}$ (D) $\frac{1}{18}$

32. **Short Response** A coin was tossed 50 times. It landed showing heads 6 more times than it landed showing tails. What is the experimental probability of the coin landing on heads? Show your work.

CHALLENGE AND EXTEND

A coin is tossed 3 times. Use the sample space for this experiment to describe each event below as *impossible, unlikely, as likely as not, likely,* or *certain.* Justify each answer.

33. At least 2 heads
34. 2 heads and 1 tail
35. 2 tails and 1 head
36. 3 tails
37. One coin is tossed 20 times.
  a. The experimental probability of the coin showing heads is 65%. How many times did the coin show tails?
  b. If the coin is tossed ten more times, how many more times must the coin land showing tails for the experimental probability of tails to be 50%?

SPIRAL REVIEW

38. A sales representative earns 4.5% commission on sales. Find the commission earned when the total sales are \$124,000. *(Lesson 2-10)*
39. Estimate the tax on a \$255 printer when the tax rate is 5.5%. *(Lesson 2-10)*

Compare the graph of each function to the graph of $f(x) = x^2$. *(Lesson 9-4)*

40. $g(x) = \frac{1}{3}x^2$
41. $g(x) = -x^2$
42. $g(x) = x^2 - 12$

The data shows the number of books read by seven students over the summer: 5, 5, 14, 2, 5, 5, 6. *(Lesson 10-3)*

43. Give the mean, median, and mode of the data.
44. Which measure of central tendency best describes the data? Explain.
45. Create a box-and-whisker plot of the data.

10-5 Technology LAB

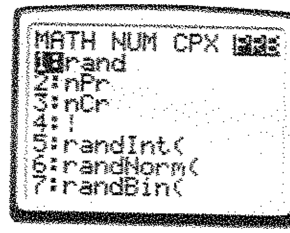
Use Random Numbers

A calculator can be used to model an experiment that would be difficult or inconvenient to perform. To do this, you will use random numbers.

Use with Lesson 10-5

Activity

You can use a calculator to explore the experimental probability that at least 2 people in a group of 6 people were born in the same month. Assume that all months are equally likely to be a person's birth month.

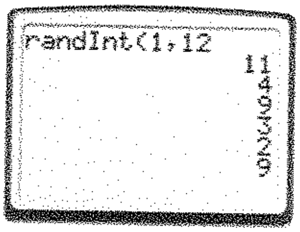


1. Represent each month with an integer. Since there are 12 months, use the numbers 1–12.

   To set your calculator up to generate random numbers, press MATH. Then use the arrow keys to highlight **PRB**. Select **5: randInt(**.

2. Now give the start number, 1, press ,, and give the end number, 12. Each time you press ENTER the calculator will return an integer from 1 to 12.

3. You are considering a group of 6 people. This means you need 6 random numbers.



	Person					
	1	2	3	4	5	6
Trial 1	11	4	9	3	2	9
Trial 2	3	8	10	12	6	2

In the first trial, the number 9 appears twice. This means that two people have a birth day in the ninth month, September.

In the second trial, no number appears more than once. This means that none of the people were born in the same month.

Try This

1. Repeat the experiment until you have 10 trials of the experiment. Count the number of trials in which a number appears more than once. Divide this number by the number of trials, 10, to find the experimental probability that at least 2 people in a group of 6 people will have the same birth month.
2. Gather the results from at least 100 trials of the experiment. (Either perform all of the trials yourself or combine data with your classmates.) Using your results, what is the experimental probability that at least 2 people in a group of 6 people will have the same birth month? Compare the results from 100 trials to the results of 10 trials.
3. How could you set up the experiment to find the experimental probability that at least 2 people in a group of 6 people will have the same birthday (same month and same date)?

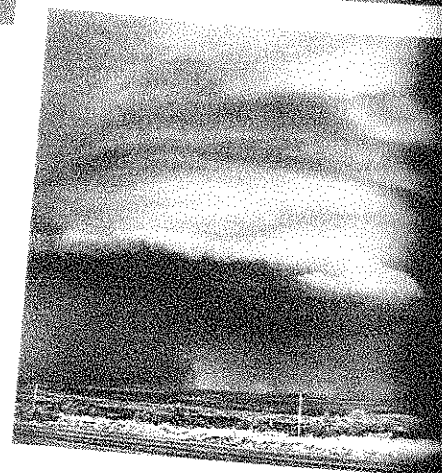
10-6 Theoretical Probability

Objectives
Determine the theoretical probability of an event.

Convert between probabilities and odds.

Vocabulary
equally likely
theoretical probability
fair
complement
odds

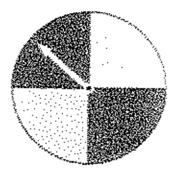
Why learn this?
Theoretical probability can be used to determine the likelihood of different weather conditions. (See Example 2.)



A developing tornado near Amarillo, Texas.

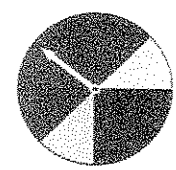
When the outcomes in the sample space of an experiment have the same chance of occurring, the outcomes are said to be **equally likely**.

Equally likely outcomes



There is the same chance that the spinner will land on any of the colors.

Not equally likely outcomes



There is a greater chance that the spinner will land on blue than on any other color.

The **theoretical probability** of an event is the ratio of the number of ways the event can occur to the total number of equally likely outcomes.

Theoretical Probability

$$\text{theoretical probability} = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

An experiment in which all outcomes are equally likely is said to be **fair**. You can usually assume that experiments involving coins and number cubes are fair.

EXAMPLE 1 Finding Theoretical Probability

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

A rolling a 3

$$\frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}} = \frac{1}{6}$$

$$= 0.1\overline{6}$$

$$= 16\frac{2}{3}\%$$

Caution!

The use of the word *experiment* does not necessarily mean you are looking for experimental probability.

An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

B rolling a number greater than 3

$$\frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}} = \frac{3}{6} = \frac{1}{2}$$ *There are 3 numbers greater than 3.*

$$= 0.5 = 50\%$$

CHECK IT OUT! An experiment consists of rolling a number cube. Find the theoretical probability of each outcome.

1a. rolling an even number **1b.** rolling a multiple of 3

Reading Math

The probability of an event can be written as $P(\text{event})$. $P(\text{heads})$ means "the probability that heads will be the outcome."

When you toss a coin, there are two possible outcomes, heads or tails. The table below shows the theoretical probabilities and experimental results of tossing a coin 10 times.

	P(heads)	P(tails)	P(heads) + P(tails)
Experimental Probability	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{3}{10} + \frac{7}{10} = \frac{10}{10} = 1$
Theoretical Probability	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

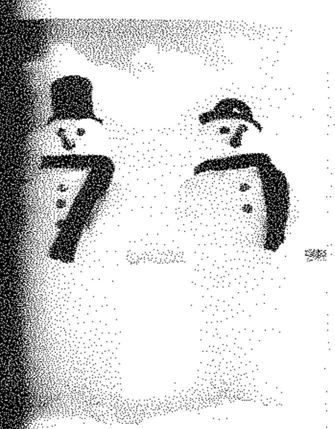
The sum of the probability of heads and the probability of tails is 1, or 100%. This is because it is certain that one of the two outcomes will always occur.

$$P(\text{event happening}) + P(\text{event not happening}) = 1$$

The **complement** of an event is all the outcomes in the sample space that are not included in the event. The sum of the probabilities of an event and its complement is 1, or 100%, because the event will either happen or not happen. Two events are *complementary events* if the sum of their probabilities equals 1 and if one event occurs if and only if the other does not.

$$P(\text{event}) + P(\text{complement of event}) = 1$$

EXAMPLE 2 Finding Probability by Using the Complement



The weather forecaster predicts a 20% chance of snow. What is the probability that it will not snow?

$$\begin{aligned} P(\text{snow}) + P(\text{not snow}) &= 100\% && \textit{Either it will snow or it will not snow.} \\ 20\% + P(\text{not snow}) &= 100\% \\ \underline{-20\%} \qquad\qquad &\quad \underline{-20\%} && \textit{Subtract 20\% from both sides.} \\ P(\text{not snow}) &= 80\% \end{aligned}$$

CHECK IT OUT! **2.** A jar has green, blue, purple, and white marbles. The probability of choosing a green marble is 0.2, the probability of choosing blue is 0.3, and the probability of choosing purple is 0.1. What is the probability of choosing white?

Odds are another way to indicate the likelihood of an event. **Odds** express likelihood by comparing the number of ways an event can happen to the number of ways an event can fail to happen. The *odds in favor of an event* describe the likelihood that the event will occur. The *odds against an event* describe the likelihood that the event will not occur.

Odds are usually written with a colon in the form $a:b$, but can also be written as a to b or $\frac{a}{b}$.

Know it! Note

Odds

ODDS IN FAVOR OF AN EVENT WITH EQUALLY LIKELY OUTCOMES

$$\text{odds in favor} = \frac{\text{number of ways an event can happen}}{\text{number of ways an event can fail to happen}} = a:b$$

ODDS AGAINST AN EVENT WITH EQUALLY LIKELY OUTCOMES

$$\text{odds against} = \frac{\text{number of ways an event can fail to happen}}{\text{number of ways an event can happen}} = b:a$$

a represents the number of ways an event can occur.

b represents the number of ways an event can fail to occur.

The two numbers given as the odds will add up to the total number of possible outcomes. You can use this relationship to convert between odds and probabilities.

EXAMPLE 3 Converting Between Odds and Probabilities

A **The probability of choosing a red card from a standard deck of playing cards is 50%. What are the odds of choosing a red card?**

The probability of choosing a red card is 50%, so the probability of not drawing a red card is 100% − 50% = 50%.

Odds in favor = 50 : 50, or 1 : 1

The odds in favor of choosing a red card are 1 : 1.

Reading Math

You may see an outcome called "favorable." This does not mean that the outcome is good or bad. A favorable outcome is the outcome you are looking for in a probability experiment.

B **The odds against choosing a green marble from a bag are 5 : 3. What is the probability of choosing a green marble?**

The odds against green are 5 : 3, so the odds in favor of green are 3 : 5. This means there are 3 favorable outcomes and 5 unfavorable outcomes for a total of 8 possible outcomes.

$$\frac{\text{number of ways event can happen}}{\text{total possible outcomes}} = \frac{3}{8}$$

The probability of choosing a green marble is $\frac{3}{8}$.

CHECK IT OUT! **3.** The odds in favor of winning a free drink are 1 : 24. What is the probability of winning a free drink?

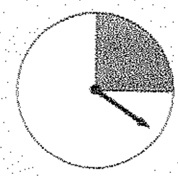
THINK AND DISCUSS

1. Tell how to find the probability of the complement of an event.

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer using the spinner.

Probabilities on Spinner	
P(gray)	
P(not gray)	
Odds in favor of gray	
Odds against gray	



10-6 Exercises

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GUIDED PRACTICE

1. **Vocabulary** All of the outcomes in the sample space that are not included in the event are called the ___?___. (*theoretical probability, complement,* or *odds*)

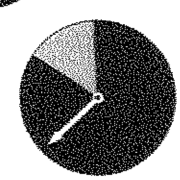
SEE EXAMPLE 1 p. 744

Find the theoretical probability of each outcome.

2. rolling a number divisible by 3 on a number cube
3. tossing 2 coins and both landing with tails showing
4. randomly choosing the letter S from the letters in STARS
5. rolling a prime number on a number cube

SEE EXAMPLE 2 p. 745

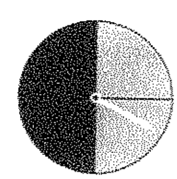
6. A spinner is green, red, and blue. The probability that the spinner will land on green is 15% and red is 35%. What is the probability the spinner will land on blue?



7. The probability of choosing a red marble from a bag is $\frac{1}{3}$. What is the probability of not choosing a red marble?
8. You have a $\frac{1}{50}$ chance of winning. What is the probability you will not win?
9. There is a $\frac{1}{10}$ chance that you will be chosen as class representative. What is the probability that you will not be chosen?

SEE EXAMPLE 3 p. 746

10. The odds against a spinner landing on blue are 3 : 1. What is the probability of the spinner landing on blue?



11. The probability of choosing an ace from a deck of cards is $\frac{1}{13}$. What are the odds of choosing an ace?
12. The probability of not winning a game is 80%. What are the odds of winning?
13. The odds in favor of a spinner landing on blue are 1 : 3. What is the probability of landing on blue?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–16	1
17–19	2
20–22	3

Extra Practice

Skills Practice p. S23

Application Practice p. S37

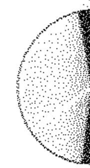
Find the theoretical probability of each outcome.

14. rolling a 5 on a number cube

15. tossing 2 coins and 1 landing with heads showing, the other with tails showing

16. randomly choosing a blue marble from a bag of 5 blue marbles, 8 red marbles, and 7 yellow marbles

17. The probability of a spinner landing on yellow is $\frac{4}{9}$. What is the probability of it not landing on yellow?

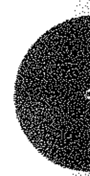


18. There is a 3% probability of winning a game. Find the probability of not winning the game.

19. There is a 15% chance it will snow and a 15% chance it will rain. What is the probability that it will neither snow nor rain?

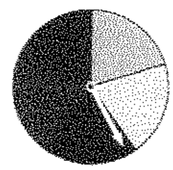
20. The odds against winning a contest are 99 : 1. What is the probability of not winning the contest?

21. The odds of choosing a white marble from a bag are 1 : 9. Find the probability of not choosing a white marble.



22. The probability of a spinner landing on green is 25%. What are the odds of the spinner not landing on green?

Use the spinner for Exercises 23–28.



23. P(red)

24. P(green)

25. P(not blue)

26. odds in favor of yellow

27. odds against red

28. odds against green

29. **Write About It** A number cube is rolled. Which event has a greater theoretical probability: rolling a number less than 3 or rolling a number greater than three? Explain.

30. **///ERROR ANALYSIS///** The odds in favor of an event are 1 : 4. Two students converted these odds into the probability of the event NOT happening. Which is incorrect? Explain the error.

Ⓐ

Odds in favor of event	1:4
Probability that event will not happen	$\frac{1}{5}$

Ⓑ

Odds in favor of event	1:4
Probability that event will not happen	$\frac{4}{5}$

31. **Critical Thinking** The odds in favor of a certain event are the same as the odds against that event. What is the probability of the event occurring?

MULTI-STEP TEST PREP

32. This problem will prepare you for the Multi-Step Test Prep on page 768.

Chutes and Ladders is a children's game that uses a spinner with the numbers 1 through 6.

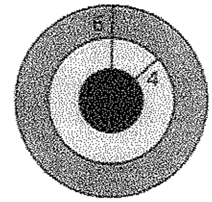
a. What is the probability of spinning a 3?

b. What is the probability of spinning an odd number?

c. What is the probability of spinning a number that is less than or equal to 4?

33. **Write About It** Explain how to convert odds to probability.

34. **Geometry** The radius of each circle in the diagram is given. Find the probability that a point chosen at random will lie in the red area of the diagram. (*Hint:* Find the ratio of the red area to the total area of the diagram.)



TEST PREP

35. Two coins are tossed. What is the probability that at least one of the coins lands with heads showing?

Ⓐ 25% Ⓑ $33\frac{1}{3}\%$ Ⓒ 50% Ⓓ 75%

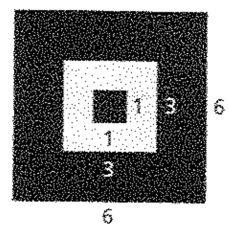
36. A standard number cube is rolled. Which has the greatest probability?

Ⓕ P(even) Ⓖ P(less than 5) Ⓗ P(not 2) Ⓙ P(greater than 3)

37. Find the probability that a point chosen at random would fall in the yellow area.

Ⓐ $\frac{1}{6}$ Ⓒ $\frac{4}{9}$

Ⓑ $\frac{2}{9}$ Ⓓ $\frac{2}{3}$



CHALLENGE AND EXTEND

Use the results of 3 coin-tossing experiments in the table for Exercise 38.

38. Find the experimental probability of the coin showing heads for

a. experiment 1.

b. experiment 2.

c. experiment 3.

d. Find the theoretical probability of the coin showing heads.

e. **Write About It** How do the experimental probabilities of each experiment compare to the theoretical probability?

	Experiment		
	1	2	3
Number of Tosses	10	100	1000
Heads	4	48	502

SPIRAL REVIEW

39. The table shows the volume of water in an office water cooler over time. Find the rate of change for each time period. For which time period did the volume of water decrease at the slowest rate? *(Lesson 5-3)*

Time of day	7:00 A.M.	9:00 A.M.	1:00 P.M.	4:00 P.M.	5:00 P.M.
Volume (gal)	4.2	3.8	2.7	1.2	0.8

Factor each trinomial. *(Lesson 8-4)*

40. $2x^2 + x - 21$ 41. $4x^2 - 7x + 3$ 42. $6x^2 + 23x + 20$

An experiment consists of choosing a card out of a deck and recording the results. Use the table to find the experimental probability of each event. *(Lesson 10-5)*

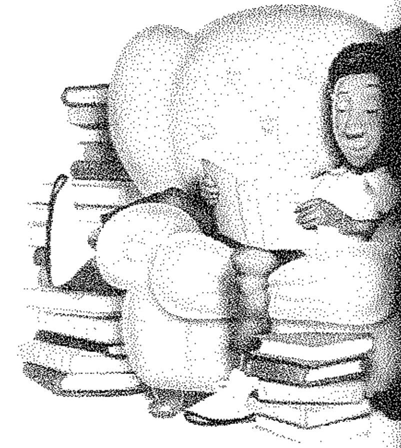
43. choosing a heart

44. choosing a heart or a diamond

45. not choosing a club

Outcome	Frequency
Hearts	2
Diamonds	6
Spades	5
Clubs	7

10-7 Independent and Dependent Events



Objectives

Find the probability of independent events.

Find the probability of dependent events.

Vocabulary

independent events
dependent events

Why learn this?

You may need to understand independent and dependent events to determine the number of reading selections available.

Adam's teacher gives the class two lists of titles and asks each student to choose two of them to read. Adam can choose one title from each list or two titles from the same list.

One title from each list

Animal Farm	Never Cry Wolf
Ethan Frome	Night
Frankenstein	Things Fall Apart
Great Expectations	Wish You Well
Jane Eyre	Wuthering Heights

Choosing a title from one list does not affect the number of titles to choose from on the other list. The events are *independent*.

Two titles from the same list

Animal Farm	~~Animal Farm~~
Ethan Frome	Ethan Frome
Frankenstein	Frankenstein
Great Expectations	Great Expectations
Jane Eyre	Jane Eyre

Choosing a title from one of the lists changes the number of titles that can be chosen from the same list. The events are *dependent*.

Events are **independent events** if the occurrence of one event does not affect the probability of the other. Events are **dependent events** if the occurrence of one event does affect the probability of the other.

EXAMPLE **1** Classifying Events as Independent or Dependent

Tell whether each set of events is independent or dependent. Explain your answer.

A **A dime lands heads up and a nickel lands heads up.**

The result of tossing a dime does not affect the result of tossing a nickel, so the events are independent.

B **You choose a colored game piece in a board game, and then your sister picks another color.**

Your sister cannot pick the same color you picked, and there are fewer game pieces for your sister to choose from after you choose, so the events are dependent.

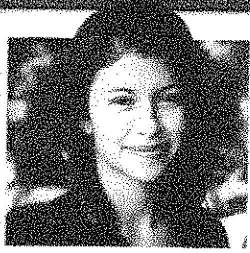
CHECK IT OUT! **Tell whether each set of events is independent or dependent. Explain your answer.**

1a. A number cube lands showing an odd number. It is rolled a second time and lands showing 6.

1b. One student in your class is chosen for a project. Then another student in the class is chosen.

Student to Student

Independent and Dependent Events



Lydia Delgado
Northridge High School

I use the everyday meanings of independent and dependent to remember their mathematical meanings.

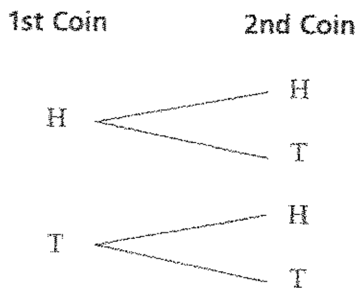
The math class I take next year depends on which math classes I have already taken.

For dependent events, the occurrence of one affects the probability of the other.

The history class I take next year is independent from (does not depend on) which math classes I have taken.

For independent events, the occurrence of one does not affect the occurrence of the other.

Suppose an experiment involves tossing two fair coins. The sample space of outcomes is shown by the tree diagram. Determine the theoretical probability of both coins landing heads up.



There are four possible outcomes in the sample space: $\{(H, H), (H, T), (T, H), (T, T)\}$

Only one outcome includes both coins landing heads up.

The theoretical probability of both coins landing heads up is $\frac{1}{4}$.

Now look back at the separate theoretical probabilities of each coin landing heads up. The theoretical probability in each case is $\frac{1}{2}$. The product of these two probabilities is $\frac{1}{4}$, the same probability shown by the tree diagram.

To determine the probability of two independent events, multiply the probabilities of the two events.

Know it! Note

Probability of Independent Events

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE 2 Finding the Probability of Independent Events

A **An experiment consists of randomly selecting a marble from a bag, replacing it, and then selecting another marble. The bag contains 7 blue marbles and 3 yellow marbles. What is the probability of selecting a yellow marble and then a blue marble?**

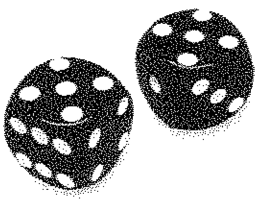
Because the first marble is replaced after it is selected, the sample space for each selection is the same. The events are independent.

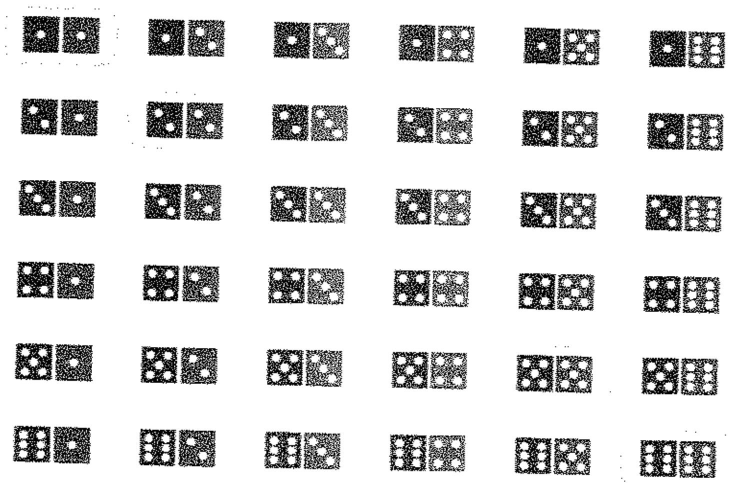
$$
\begin{aligned}
P(\text{yellow, blue}) &= P(\text{yellow}) \cdot P(\text{blue}) \\
&= \frac{3}{10} \cdot \frac{7}{10} \qquad \textit{The probability of selecting yellow is } \tfrac{3}{10}\textit{, and the probability of selecting blue is } \tfrac{7}{10}\textit{.} \\
&= \frac{21}{100}
\end{aligned}
$$

B **When a person rolls 2 number cubes and they land showing the same number, we say the person rolled doubles. What is the probability of rolling doubles 3 times in a row?**

The result of one roll does not affect any following rolls. The events are independent.

When you roll 2 number cubes, there are 36 possible outcomes, six of which are doubles:

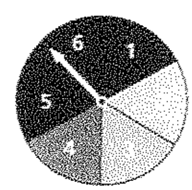




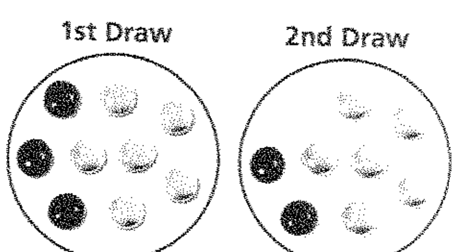
So, the probability of rolling doubles once is $P(\text{double}) = \frac{6}{36} = \frac{1}{6}$.

$$P(\text{double, double, double}) = P(\text{double}) \cdot P(\text{double}) \cdot P(\text{double})$$
$$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$$
$$= \frac{1}{216}$$

CHECK IT OUT! **2.** An experiment consists of spinning the spinner twice. What is the probability of spinning two odd numbers?



Suppose an experiment involves drawing marbles from a bag. Determine the theoretical probability of drawing a red marble and then drawing a second red marble *without replacing the first one.*



The sample space for the second draw is not the same as the sample space for the first draw. There are fewer marbles in the bag for the second draw.

This means the events are dependent.

Probability of drawing a red marble on the first draw $= \frac{3}{9} = \frac{1}{3}$

Probability of drawing a red marble on the second draw $= \frac{2}{8} = \frac{1}{4}$

To determine the probability of two dependent events, multiply the probability of the first event times the probability of the second event after the first event has occurred.

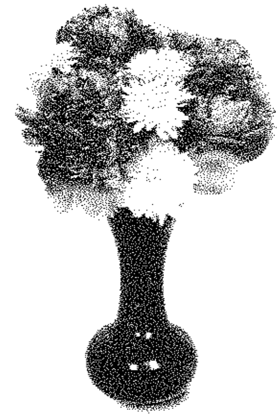
Know it! Note

Probability of Dependent Events

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$.

EXAMPLE 3 *Problem-Solving Application*

PROBLEM SOLVING



There are 7 pink flowers and 5 yellow flowers in a bunch. Jane selects a flower at random, and then Leah selects a flower at random from the remaining flowers. What is the probability that Jane selects a pink flower and Leah selects a yellow flower?

1 Understand the Problem

The **answer** will be the probability that a yellow flower is chosen after a pink flower is chosen.

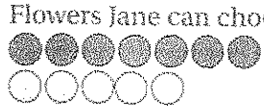
List the **important information:**

- Jane chooses a pink flower from 7 pink flowers and 5 yellow flowers.
- Leah chooses a yellow flower from 6 pink flowers and 5 yellow flowers.

2 Make a Plan

Draw a diagram.

Flowers Jane can choose from:



7 pink
5 yellow
12 total

Flowers Leah can choose from:

6 pink
5 yellow
11 total

After Jane selects a flower, the sample space changes. So the events are dependent.

3 Solve

$$P(\text{pink and yellow}) = P(\text{pink}) \cdot P(\text{yellow after pink})$$

$$= \frac{7}{12} \cdot \frac{5}{11}$$

Jane selects one of 7 pink flowers from 12 total flowers. Then Leah selects one of 5 yellow flowers from the 11 flowers left.

$$= \frac{35}{132}$$

The probability that Jane selects a pink flower and Leah selects a yellow flower is $\frac{35}{132}$.

4 Look Back

Drawing a diagram helps you see how the sample space changes. This means the events are dependent, so you can use the formula for probability of dependent events.

CHECK IT OUT! 3. A bag has 10 red marbles, 12 white marbles, and 8 blue marbles. Two marbles are randomly drawn from the bag. What is the probability of drawing a blue marble and then a red marble?

THINK AND DISCUSS

1. Give an example of two events that are dependent. Explain why the events are dependent.

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer.

	Example	Probability
Dependent Events	$A =$ $B =$	$P(A \text{ and } B) =$
Independent Events	$A =$ $B =$	$P(A \text{ and } B) =$

10-7 Exercises

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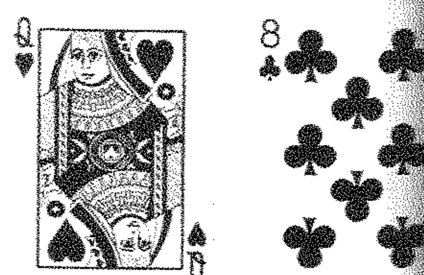
GUIDED PRACTICE

1. **Vocabulary** Two events are ___?___ if the occurrence of one event affects the probability of the other event. (*independent* or *dependent*)

SEE EXAMPLE 1 p. 750

Tell whether each set of events is independent or dependent. Explain your answers.

2. You draw a heart from a deck of cards and set it aside. Then you draw a club from the deck of cards.



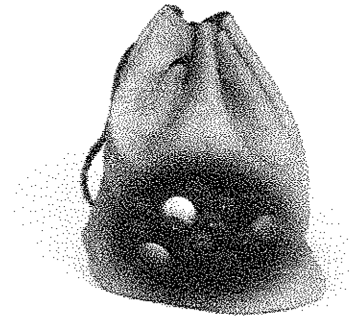
3. You guess "true" on two true-false questions.
4. You select a kitten to adopt from an animal shelter. Then your friend selects a kitten to adopt.
5. You order from a menu, and then your friend orders.
6. A doctors' office schedules several patients. Then you make an appointment.

SEE EXAMPLE 2 p. 751

7. A coin is tossed three times. What is the probability of the coin landing heads up three times?
8. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two odd numbers?
9. Stacey rolls two number cubes. What is the probability that the sum of the numbers on the two number cubes is 7?
10. A number cube is rolled twice and a coin is tossed once. What is the probability of the coin landing heads up and the number cube landing with 2 showing both times?
11. A spinner with four equal sections of red, yellow, green, and blue is spun twice. Wha is the probability that it lands on yellow and then on green?

EXAMPLE 3
p. 753

12. A bag contains 4 red marbles, 3 white marbles, and 6 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting a white marble from the bag?



13. Seven cards are numbered from 1 to 7 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two odd numbers?

14. There are 15 boys and 14 girls in a room. Two of them are selected at random to take a survey. What is the probability that the two people selected will be girls?

PRACTICE AND PROBLEM SOLVING

...ndent Practice

For ...rcises	See Example
...–16	1
...–19	2
...–22	3

...ra Practice
...ractice p. S23
...tion Practice p. S37

Tell whether each set of events is independent or dependent. Explain your answer.

15. The teacher randomly selects two students from the class.

16. You roll a 3 on a number cube and choose a 3 from a deck of cards.

17. A number cube is rolled three times. What is the probability of rolling three even numbers?

18. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and replaced. Another card is randomly selected. What is the probability of selecting two even numbers?

19. Stacey rolls a number cube and tosses a coin. What is the probability that she rolls a 5 and the coin lands heads up?

20. A bag contains 5 red marbles, 3 white marbles, and 4 blue marbles. What is the probability of randomly selecting a red marble, setting it aside, and then randomly selecting another red marble from the bag?

21. Ten cards are numbered from 1 to 10 and placed in a box. One card is selected at random and not replaced. Another card is randomly selected. What is the probability of selecting two even numbers?

22. A game has 6 colored playing pieces. They are red, yellow, green, blue, purple, and white. You and a friend each pick a game piece without looking. What is the probability that your friend picks the blue piece and then you pick the yellow piece?

LINK
...ool

...me standardized ...there is no penalty ...wrong answer. On ...tests, it is better to ...than to leave the ...r blank, especially ...e choices can be ...ated.

23. **School** On a multiple-choice test, each question has 4 possible answers. A student does not know the answers to three questions, so the student guesses.

a. What is the probability that the student gets all three questions wrong?

b. What is the probability that the student gets all three questions right?

Tell whether each set of events is independent or dependent. Explain your answer.

24. Pick "Joe" from a box of names, replace it, and then pick "Craig."

25. Pick "Joe" from a box of names, set it aside, and then pick "Craig."

26. Roll a prime number on a number cube and get tails when tossing a coin.

27. Roll an even number, then an odd number, and then a 1 on a number cube.

MULTI-STEP TEST PREP

28. This problem will prepare you for the Multi-Step Test Prep on page 768.

Yahtzee is a game that involves rolling five dice. On his or her turn, a player ca up to three times to try to score points in various categories. Rolling a "Yahtzee" means rolling five of a kind, or five of the same number.

a. Juan has rolled twice and has three 5's showing. He rolls the remaining two d What is the probability that both dice will land showing 5?

b. Shauna has two 3's showing. She has one more roll with the remaining three dice. What is the probability that all three dice will land showing 3?

c. Mike rolls all five number cubes and all of them land showing 6. What is the probability of getting five 6's in one roll?

29. A bag contains 3 red, 5 blue, and 2 white marbles.

a. Find the probability of randomly picking a red marble, replacing it, and then picking a blue marble.

b. Find the probability of randomly picking a red marble, setting it aside, and then picking a blue marble.

c. Find the probability of randomly picking a red marble, replacing it, and then picking another red marble.

d. Find the probability of randomly picking a red marble, setting it aside, and then picking another red marble.

30. **Entertainment** Joe and Maria are playing a board game. On each turn, the player rolls two number cubes. Both players have two turns remaining.

a. Joe will win if he rolls double 6's on both turns. What is the probability that Joe will roll double 6's on both turns?

b. Maria will win if she rolls 2 on the first turn and 12 on the second turn. What is the probability that Maria will roll 2 on the first turn and 12 on the second turn?

c. **Write About It** Who has the better probability of winning? Explain.

31. Tamika has $2.50 in quarters in her pocket, including four state quarters. She reaches into her pocket and takes out two quarters, one at a time. What is the probability that they are both state quarters?

32. Ten cards are numbered 1 through 10 and placed in a bag. You draw a card, set it aside, and draw another card. What is the probability that you will draw two numbers that are divisible by 3?

33. **Critical Thinking** What is the probability of a coin landing heads up on two toss if it lands tails up on the first toss? Explain.

34. **Write About It** Explain what it means for two events to be independent.

TEST PREP

35. A number cube is rolled twice. What is the probability of getting a 2 on both rol

Ⓐ $\frac{1}{3}$ Ⓑ $\frac{1}{4}$ Ⓒ $\frac{1}{9}$ Ⓓ $\frac{1}{36}$

36. In baseball, Julio averages 3 hits in every 10 at bats. What is the probability that he will get hits in both of his next two at bats?

Ⓕ 0.03 Ⓖ 0.09 Ⓗ 0.3 Ⓙ 0.9

37. Two people from a group of 30 will be selected at random for a prize. Twenty people in the group are women. What is the probability that both people selected will be men?

Ⓐ $\frac{3}{29}$ Ⓑ $\frac{38}{87}$ Ⓒ $\frac{56}{87}$ Ⓓ $\frac{1}{10}$

38. Ravi has 10 pairs of socks in a drawer, but none of the pairs are matched up. Each pair is a different color, and one pair is blue. Ravi has to pick his socks in the dark so he does not wake his brother. Which expression can be used to find the probability that Ravi will choose a blue sock and then the matching sock?

Ⓕ $\frac{1}{20} + \frac{1}{20}$ Ⓖ $\frac{1}{10} \cdot \frac{1}{20}$ Ⓗ $\frac{1}{20} + \frac{1}{19}$ Ⓙ $\frac{1}{10} \cdot \frac{1}{19}$

CHALLENGE AND EXTEND

39. **Basketball** Terrance has made 90% of the free throws he has attempted at basketball practice. What is the probability that he will make the next three free throws he attempts?

40. A number cube is rolled three times. What is the probability of rolling a 5 at least once?

Geometry Use the grid for Exercises 41–44.

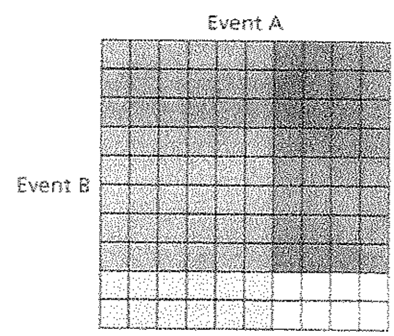
On the grid at right, one small square represents 1% probability.

The shaded rows represent the probability that event *A* occurs.

The shaded columns represent the probability that event *B* occurs.

The area where the shaded rows and columns overlap represents the probability that both events occur.

Use the grid to find each probability.



41. Event *A* occurs.

42. Event *B* occurs.

43. Event *A* occurs AND event *B* occurs.

44. Neither event *A* nor event *B* occurs.

SPIRAL REVIEW

45. A tennis player serves 2 aces (unreturned serves) for every 17 serves. If the player serves 204 times in the next match, how many serves would you expect to be aces? *(Lesson 2-7)*

Compare the graph of each function to the graph of $f(x) = x^2$. *(Lesson 9-4)*

46. $g(x) = 4x^2 - 1$ 47. $g(x) = \frac{1}{5}x^2$ 48. $g(x) = -x^2 + 3$

Find the theoretical probability of each outcome. *(Lesson 10-6)*

49. Randomly selecting a blue marble out of a bag with 6 red and 9 blue marbles

50. Rolling a number less than 10 on a number cube

51. Randomly selecting A, E, I, O, or U from all letters of the alphabet

10-7
Algebra LAB

Use with Lesson 10-7

Compound Events

When two events cannot happen at the same time, they are called *mutually exclusive* events. When two events can happen at the same time, they are called *inclusive* events. You can use sample spaces to determine the probabilities of mutually exclusive events and inclusive events.

When finding the probablity of a compound event, you should first determine whether the simple events involved are dependent or independent.

Activity 1

Suppose you are playing Monopoly and are "just visiting" the Jail. If you roll 7 on your next turn, you will land on Community Chest. If you roll 12, you will land on Chance. What is the probability that on your next roll you land on either Community Chest or Chance?

You cannot land on Community Chest and Chance at the same time, so the events are mutually exclusive. You cannot roll 7 and 12 at the same time, so those events are also mutually exclusive.

The table shows some of the totals when rolling two fair dice. Copy and complete the table. Use one color to circle all rolls with a total of 7. Use a second color to circle all rolls with a total of 12.

+	1	2	3	4	5	6
1	2	3	4	5		
2	3	4				
3	4	5				
4	5					
5	6					
6	7					

Try This

1. What is the total number of possible rolls?
2. What is the probability that the total will be 7?
3. What is the probability that the total will be 12?
4. What is the probability that the total will be 7 or 12?
5. What do you notice about the probabilities in Problems 2, 3, and 4?

Suppose you are three spaces away from Community Chest and eight spaces away from Chance.

6. What is the probability that on your next roll the total will be 3?
7. What is the probability that on your next roll the total will be 8?
8. What is the probability that on your next roll the total will be 3 or 8?
9. Complete the following statement:

   The probability that one of two mutually exclusive events will occur is the ___?___ of the probabilities of the individual events.

In this lab, you discovered this rule: If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.

Activity 2

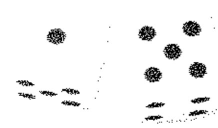
Suppose you are playing Yahtzee and on the last roll of your turn, you roll two of the five dice. You need to roll either a 1 or a 5 to make what is called a "small straight" (four consecutive numbers).

What is the probability that you will make a small straight?

You can roll a 1 and a 5 at the same time on different dice. These events are inclusive.

(If you roll a 1 and a 5, you will need to look at only four of the dice to make a small straight.)

There are three outcomes that involve rolling a 1 or rolling a 5.

Case 1	Case 2	Case 3
		
At least one 1	At least one 5	Both a 1 and a 5

Try This

10. Find the probability of Case 1, rolling at least one 1.
    a. What is the probability of rolling ⚀ on the first die and any number on the second die?
    b. What is the probability of rolling any number on the first die and ⚀ on the second die?
    c. What is the probability of rolling ⚀ on the first die and ⚀ on the second die?
    d. Parts a and b both include rolling ⚀ on the first die and ⚀ on the second die. You need to count this outcome only once, so subtract the probability from part c from the sum of parts a and b.
11. Find the probability of Case 2, rolling at least one 5. (*Hint:* Repeat the steps in Problem 10.)
12. Find the probability of Case 3, rolling a 1 and a 5.
13. The probability of rolling a small straight, is the probability of rolling ⚀ *or* ⚄.

    Subtract the probability of rolling both ⚀ *and* ⚄ from the sum of the probabilities of rolling ⚀ on one of the dice (Problem 10) and rolling ⚄ on one of the dice (Problem 11).
14. Complete the following statement:

    The probability that one of two inclusive events will occur is the __?__ of the probabilities of the individual events __?__ the probability of the intersection of the two events.

In this lab, you discovered this rule:

If A and B are inclusive events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

10-8 Combinations and Permutations

Solve problems involving permutations.

Solve problems involving combinations.

Vocabulary
compound event
combination
permutation

Security companies use permutations to create many security codes using a limited number of digits or letters. (See Exercise 8.)

Sometimes there are too many possible outcomes to make a tree diagram or a list. The *Fundamental Counting Principle* is one method of finding the number of possible outcomes.

Know it! Note

Fundamental Counting Principle

If there are m ways to choose a first item and n ways to choose a second item after the first item has been chosen, then there are $m \cdot n$ ways to choose both items.

The Fundamental Counting Principle can also be used when there are more than two items to choose.

EXAMPLE 1 Using the Fundamental Counting Principle

A florist is arranging centerpieces that include 1 flower, 1 plant, and 1 vase. The florist has 2 kinds of vases, 2 kinds of plants, and 3 kinds of flowers to choose from. How many different centerpieces are possible?

Method 1 Use a tree diagram.

There are 12 possible centerpieces.

Method 2 Use the Fundamental Counting Principle.

$2 \cdot 2 \cdot 3$

12

There are 12 possible centerpieces.

CHECK IT OUT! 1. A voicemail system password is 1 letter followed by a 3-digit number less than 600. How many different voicemail passwords are possible?

A **compound event** consists of two or more simple events, such as a rolled number cube landing with 3 showing and a tossed coin landing heads up. (A simple event has only one outcome, such as rolling a 3 on a number cube.) For some compound events, the order in which the simple events occur is important.

A **combination** is a grouping of outcomes in which the order does not matter. A **permutation** is an arrangement of outcomes in which the order does matter.

EXAMPLE 2 Finding Combinations and Permutations

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

A **A street vendor sells cashews, peanuts, and almonds. How many different ways are there to mix two kinds of nuts?**

List all of the possible groupings:

cashews and peanuts	peanuts and almonds	almonds and cashews
peanuts and cashews	almonds and peanuts	cashews and almonds

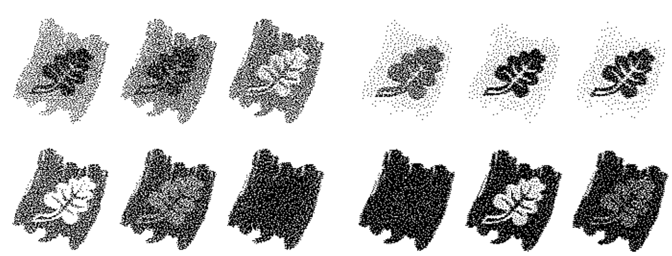
The order of the outcomes is not important, so this situation involves combinations. Eliminate the groupings that are duplicates.

cashews and peanuts	peanuts and almonds	almonds and cashews
~~peanuts and cashews~~	~~almonds and peanuts~~	~~cashews and almonds~~

There are 3 different ways to mix two kinds of nuts.

B **Karen is painting her bedroom. She has orange, green, blue, and purple paint. She plans to use one color as a base coat and stencil a design with another color. How many different ways can she do this?**

List all of the possible groupings:



The order of the outcomes is important, because one color is painted on top of the other. This situation involves permutations.

There are 12 different ways for Karen to paint her room.

CHECK IT OUT! **Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.**

2a. Ingrid is stringing three different types of beads on a bracelet. How many ways can she use one bead of each type to string the next three beads?

2b. Nathan wants to order a sandwich with two of the following ingredients: mushroom, eggplant, tomato, and avocado. How many different sandwiches can Nathan choose?

Helpful Hint

The factorial of 0 is defined to be 1.
$0! = 1$

The factorial of a number is the product of the number and all the natural numbers less than the number. The factorial of 5 is written 5! and is read "five factorial." $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. Factorials can be used to find the number of combinations and permutations that can be made from a set of choices.

Suppose you want to make a five-letter password from the letters *A, B, C, D,* and *E* without repeating a letter. You have 5 choices for the first letter, but only 4 choices for the second letter. You have one fewer choice for each letter of the password.

First letter   Second letter   Third letter   Fourth letter   Fifth letter

5 choices × 4 choices × 3 choices × 2 choices × 1 choice = 120 permutations

Suppose you want to make a three-letter password from the 5 letters *A, B, C, D,* and *E* without repeating a letter. Again, you have one fewer choice for each letter of the password.

First letter   Second letter   Third letter

There are 5 choices (A, B, C, D, E), and you are choosing 3 of them.

5 choices × 4 choices × 3 choices = 60 permutations

The number of permutations is $\frac{5!}{2!}$, or $\frac{5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 5 \cdot 4 \cdot 3 = 60$.

Know it! Note

Permutations

FORMULA	The number of permutations of n things chosen r at a time: $\quad {}_nP_r = \frac{n!}{(n-r)!}$
EXAMPLE	A club will choose a president, a vice president, and a secretary from a list of 8 people. How many ways can the club choose the 3 officers? The position that each person takes matters, so this situation involves permutations. Think: There are 8 people, and the club will choose 3 of them. $\quad {}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$

EXAMPLE 3 Finding Permutations

Lee brings 7 CDs numbered 1–7 to a party. How many different ways can he choose the first 4 CDs to play?

The order in which the CDs are played matters, so use the formula for permutations.

$${}_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!}$$
$n = 7$ and $r = 4$.

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$
A number divided by itself is 1, so you can divide out common factors in the numerator and denominator.

$$= 7 \cdot 6 \cdot 5 \cdot 4 = 840$$

There are 840 different ways Lee can choose to play the 4 CDs.

CHECK IT OUT! 3. How many different ways can 9 people line up for a picture?

The formula for combinations also involves factorials.

Know it! *Note*

Combinations

FORMULA	The number of combinations of n things chosen r at a time: $\quad {}_nC_r = \dfrac{n!}{r!(n-r)!}$.
EXAMPLE	A club will form a 3-person committee from a list of 8 people. How many ways can the club choose the 3 people? The position that each person takes does not matter, so this situation involves combinations. Think: There are 8 people, and the club will choose 3 of them. $\quad {}_8C_3 = \dfrac{8!}{3!(8-3)!} = \dfrac{8!}{3!5!} = 56$

EXAMPLE 4 Finding Combinations

There are 11 different cereals for sale at a grocery store. How many different ways can a shopper select 4 different cereals?

The order in which the cereals are selected does not matter, so use the formula for combinations.

Method 1 Use the formula for combinations.

$$ {}_{11}C_4 = \frac{11!}{4!\,(11-4)!} = \frac{11!}{4!7!} \qquad n = 11 \text{ and } r = 4. $$

$$ = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(4 \cdot 3 \cdot 2 \cdot 1)(\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1})} $$

$$ = \frac{7920}{24} = 330 $$

Method 2 Use the **nCr** function of a calculator.

```
11 nCr 4
             330
```

There are 330 different ways the shopper can choose 4 different cereals.

Helpful Hint

You can also use a calculator to find permutations. Look for the **nPr** function on your calculator.

CHECK IT OUT! 4. A basketball team has 12 members who can play any position. How many different ways can the coach choose 5 starting players?

THINK AND DISCUSS

1. Explain how to find $\frac{10!}{9!}$ without a calculator.

Know it! *Note*

2. GET ORGANIZED Copy and complete the graphic organizer.

	Fundamental Counting Principle	Permutation	Combination
When to Use			
Formula			

10-8 Exercises

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Homework Help On
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Parent Resources
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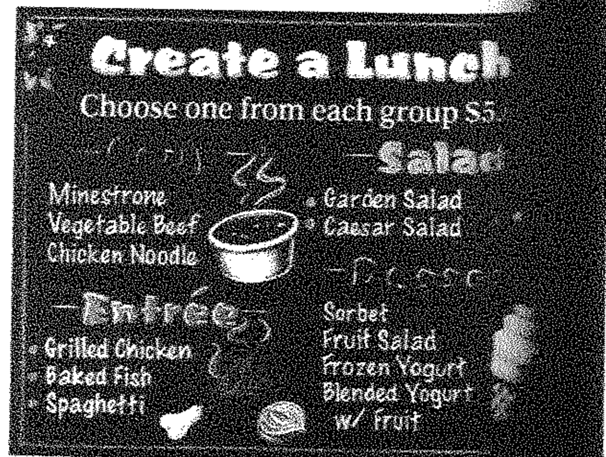
GUIDED PRACTICE

1. **Vocabulary** A ___?___ is an arrangement of outcomes in which the ord does not matter. (*compound event*, *permutation*, or *combination*)

SEE EXAMPLE 1 p. 760

2. The menu for a restaurant is shown. How many different meals with one salad, one soup, one entree, and one dessert are possible?



3. You have 4 colors of wrapping paper and 2 kinds of ribbon. How many different ways are there to use one color of paper and one type of ribbon to wrap a package?

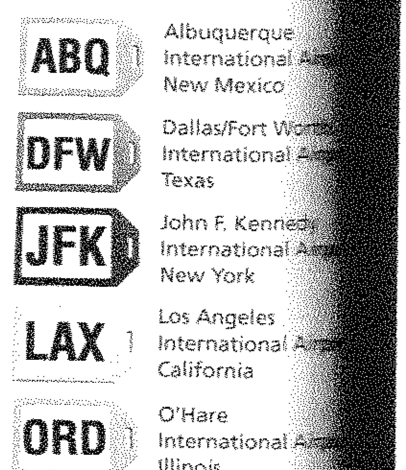
SEE EXAMPLE 2 p. 761

Tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

4. How many different ways can 4 people be seated in a row of 4 seats?
5. How many different kinds of punch can be made from 2 of the following: cranbe juice, apple juice, orange juice, and grape juice?

SEE EXAMPLE 3 p. 762

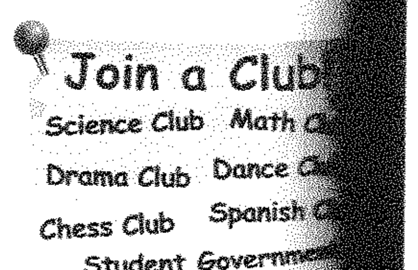
6. An airport identification code is made up of three letters, like the examples shown. How many different airport identification codes are possible? (Assume any letter can be repeated and can appear in any position.)



7. A manager is scheduling interviews for job applicants. She has 8 time slots and 6 applicants to interview. How many different interview schedules are possible?
8. An e-mail server password must be 6 different lowercase letters. How many different e-mail server passwords are possible?

SEE EXAMPLE 4 p. 763

9. Francine is deciding which after-school clubs to join. She has time to participate in 3. How many different ways can Francine choose which 3 clubs to join?



10. Laura is making gift baskets. Each basket has 2 gifts, and she has 5 gifts to choose from. How many different gift baskets can Laura make?
11. How many different ways can a contest judge select 5 finalists from 20 contestants?

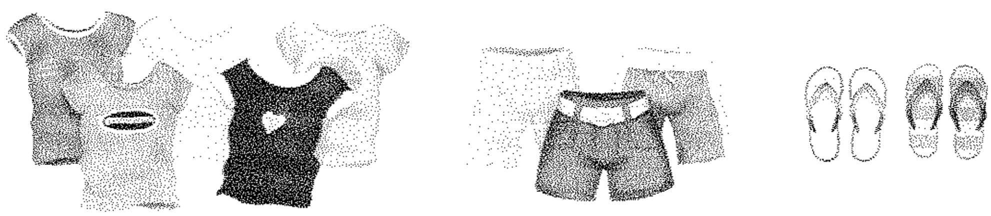
...ndent Practice

...or ...ercises	See Example
...2–13	1
...–15	2
...–17	3
...–19	4

...ra Practice

...ractice p. 523
...tion Practice p. 537

12. Maria looks in her closet and exclaims, "I have nothing to wear!" How many different outfits of one shirt, one pair of shorts, and one pair of sandals are possible using the items shown?

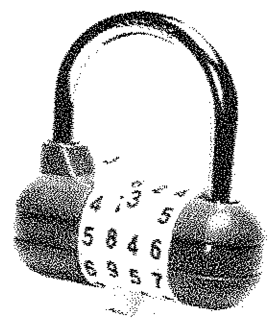


13. When a customer buys tickets for a concert, the ticket office assigns a confirmation code that is made up of 2 lowercase consonants, followed by a 3-digit number from 000 to 999. A letter or digit may be repeated. How many different confirmation codes are possible?

For Exercises 14 and 15, tell whether each situation involves combinations or permutations. Then give the number of possible outcomes.

14. A team of archeologists divides a dig site into 3 areas. They dig one area at a time. How many different ways can they order the 3 areas?

15. To decide which team will lead the class discussion, a teacher writes the names of 5 students on slips of paper and puts them in a hat. Then she draws 2 names. How many teams of two are possible?



16. The code for a bicycle lock is made up of 4 digits from 0 through 9. How many different codes are possible?

17. A television station has 5 different commercials to play during the news. In how many different ways can they order the commercials?

18. David's summer reading list has 9 books. How many different ways can David select 3 books to read?

19. Steve draws a hand of 7 cards from a deck of 52 different cards. How many different hands are possible?

LINK

...tory

...one "numbers" ...n be made up of ...etters and several ... Calls were ... by speaking the ...one number to a ... operator, who ... connected the call ...d.

20. **History** The North American Numbering Plan (NANP) was first used in 1947. It is a system for assigning telephone numbers and area codes.

   a. Originally, the NANP allowed only 3-digit area codes whose first digit was not 0 or 1, and whose second digit was always 0 or 1. How many different area codes were possible under this system?

   b. In 1995, because of increased demand, the NANP removed the restriction that the second digit of the area code must be 0 or 1. How many more area codes did this make possible?

21. **Critical Thinking** For $n = 6$ and $r = 2$, which is larger: ${}_nP_r$ or ${}_nC_r$? Explain why this is true.

22. **Write About It** Explain why a "combination lock" should be called a "permutation lock."

23. **Write About It** You roll a number cube 6 times. Explain how to determine the number of possible outcomes. Would a tree diagram be useful in solving this problem? Explain why or why not.

MULTI-STEP TEST PREP

24. This problem will prepare you for the Multi-Step Test Prep on page 768.
Many states have lotteries to raise money. In one lottery, the players choose 6 different numbers from the numbers 1–49. The order of the numbers does not matter.
   a. How many combinations are there?
   b. The probability that your first number is on the winning ticket is $\frac{6}{49}$. The probability that your second number matches is $\frac{5}{48}$. The probabilities of the third, fourth, fifth, and sixth numbers matching are $\frac{4}{47}$, $\frac{3}{46}$, $\frac{2}{45}$, and $\frac{1}{44}$. The product of these probabilities is the probability of winning the lottery with one ticket. Find that probability. How does it relate to the number of combinations shown in part a?

25. **Technology** Nancy's password for her online banking program is made up of 6 letters. It does not matter whether the letters are uppercase or lowercase and any letter may be repeated.
   a. What is the probability of someone correctly guessing Nancy's password?
   b. How many hours would it take for someone to try all the possible passwords if the person could guess one password each second?
   c. **Write About It** Explain why you think some Internet companies require that passwords be at least 6 characters long.

TEST PREP

26. Which situation is best modeled by the expression ${}_{10}C_4$?
   (A) Lisa's salon offers 10 kinds of shampoo and 4 kinds of conditioner. How many different ways can she shampoo and condition a client's hair?
   (B) Sara has a box of 10 colored pencils. How many different ways can she choose 4 colored pencils from the box?
   (C) A personal identification number (PIN) is a 4-digit number. Digits can be repeated in a PIN. How many different PINs are possible?
   (D) There are 10 chairs placed around a table. How many different ways can 4 people seat themselves around the table?

27. Which of the following is equal to 4!?
   (F) $10! - 6!$ (G) $\frac{8!}{2!}$ (H) $(2!)(2!)$ (J) $\frac{24!}{23!}$

28. Which table represents the values of ${}_xC_4$ for the given values of x?

(A)

x	${}_xC_4$
5	5
6	30
7	210
8	1680

(B)

x	${}_xC_4$
5	120
6	360
7	840
8	1680

(C)

x	${}_xC_4$
5	5
6	15
7	35
8	70

(D)

x	${}_xC_4$
5	30
6	90
7	210
8	240

29. **Short Response** The periodic table of the elements is a chart listing 117 different elements. Maria must give a presentation on 3 different elements from the periodic table. How many ways can she choose which elements to present? Show your work and explain each step.

CHALLENGE AND EXTEND

Simplify each expression.

30. $\frac{8!}{3!}$ **31.** $\frac{7!4!}{6!}$ **32.** $\frac{{}_7P_5}{{}_7P_3}$

33. The chart shows the different toppings offered by a pizza shop.

Pizza Ingredients

- Extra Cheese
- Green Peppers
- Mushrooms
- [illegible]
- Onions
- Pineapple
- Green Olives

a. How many different pizzas with 2 toppings are possible?

b. How many different pizzas with 6 toppings are possible?

c. What do you notice about your answers to parts **a** and **b**? Explain why you think this is.

34. Use the permutation and combination formulas you learned in this lesson.

a. What is the number of permutations when $n = r$?

b. What is the number of combinations when $n = r$?

35. You roll a number cube six times in a row. What is the probability that you roll the numbers 1 through 6 in order?

SPIRAL REVIEW

Simplify each expression. *(Lesson 1-7)*

36. $4.9m + 3.8 - 2.1m - 3.9$ **37.** $4(2x - 1) + 9 - 3x$ **38.** $3a^2 - 5a - 7a^2$

Identify the independent and dependent variables. Write an equation in function notation for each situation. *(Lesson 4-3)*

39. A bathtub fills at a rate of 15 gallons per minute.

40. About 5 seeds should be planted per square inch.

41. A snow removal company charges a contract fee of $300 plus $80 per hour.

42. Five people responded that they spend the following amounts for haircuts during one year: $150, $120, $135, $0, and $145. A researcher concluded that "people spend an average of $110 per year on haircuts." Why is this statistic misleading? *(Lesson 10-4)*

Career Path

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KEYWORD: MA7 Career

Erika Sheehan
Biostatistics major

Q: What math classes did you take in high school?

A: Algebra, Geometry, Precalculus, Calculus

Q: What math classes have you taken in college?

A: Statistics, Linear Algebra, Problem Solving

Q: What do Biostatisticians do?

A: Biostatisticians apply statistical methods to research scientific questions relating to health and medicine.

Q: What are your plans for the future?

A: After graduating, I would like to get a job at a pharmaceutical company testing the safety and effectiveness of new medicines.